A Tour Up The Gray Scale Vector of the RGB Color Cube: How Computer Graphics Color Spaces Relate to Digital Video Color Difference Space

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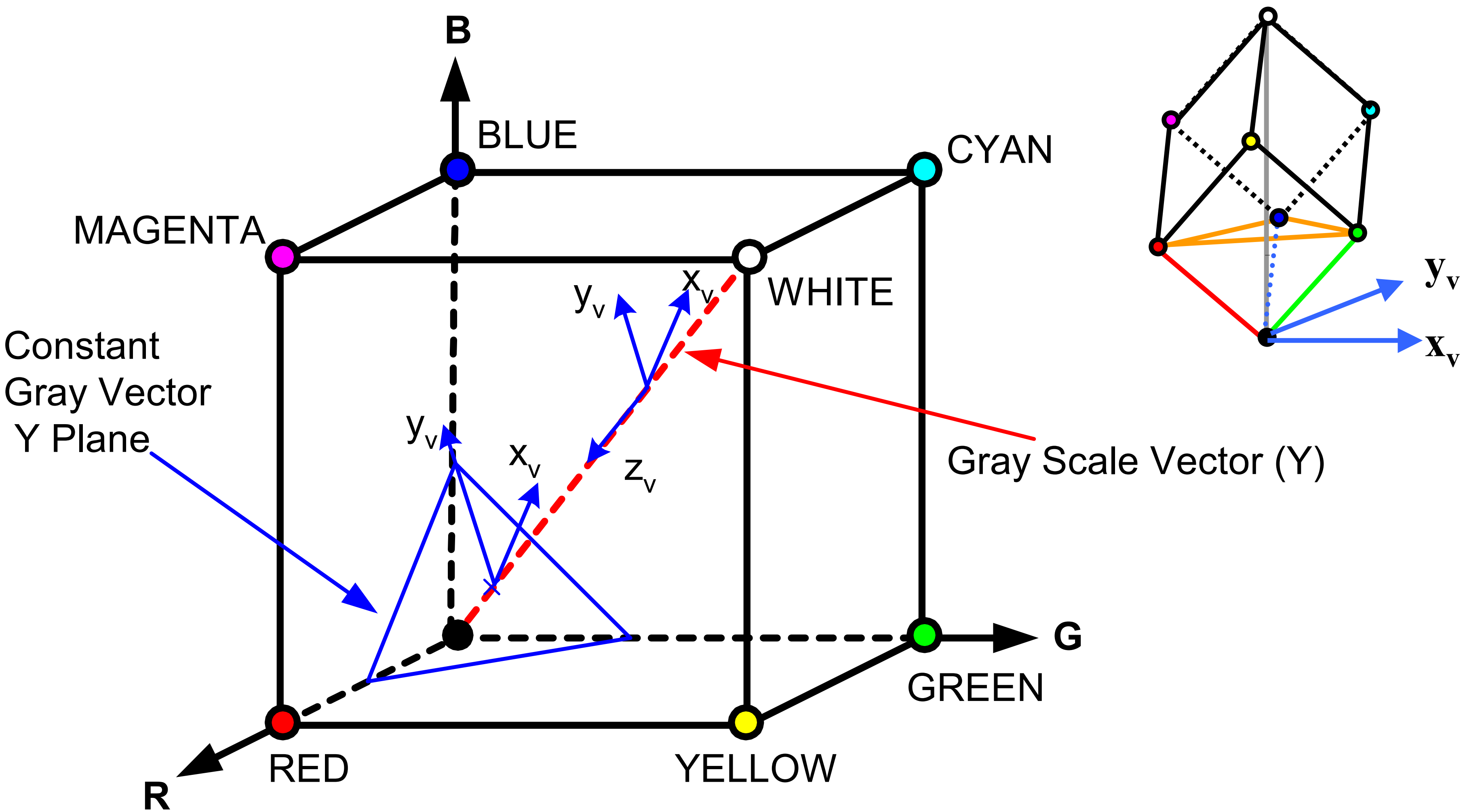
What I Will Talk About

- Show various color slices within RGB color cube
  - **Tour up the gray vector showing orthogonal color planes!!**
  - Three intersecting planes that bisect the gray scale vector
- HSV (or HLS) color space
  - Color cross sections of HSV space
  - How HSV (or HLS) relate to RGB
- Component Video Color Difference Space
  - **Tour up the luminance vector!!**
  - Recommendation 601 standard
  - Interesting geometric relationship to RGB

The Tour: RGB Unit Color Cube

Showing (x_v,y_v,z_v) View Coordinate System



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The Tour: Formulation

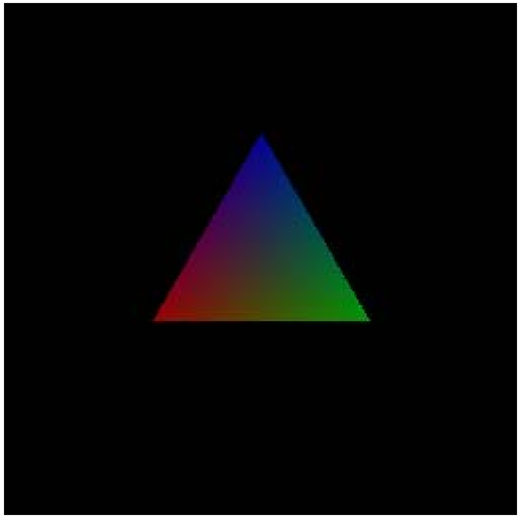
Using homogeneous coordinates to rotate and translate from RGB space to Yx_vy_v space yields

$$\begin{bmatrix} Y \\ x_v \\ y_v \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \end{bmatrix} * \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ -0.707 & 0.707 & 0 \\ -0.408 & -0.408 & 0.816 \end{bmatrix} * \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

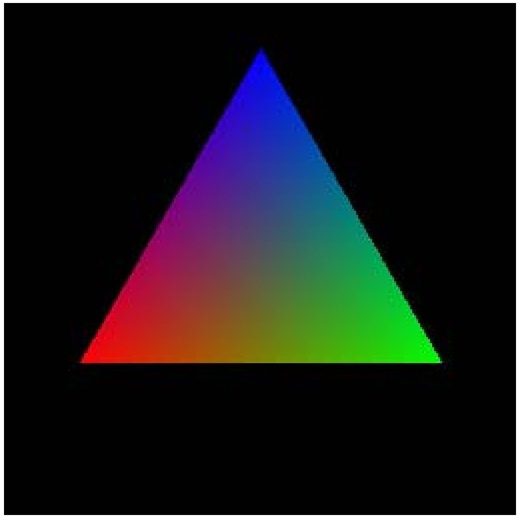
Solving for RGB so we can generate color planes yields

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 0.577 & -0.707 & 0.408 \\ 0.577 & 0.707 & -0.408 \\ 0.577 & 0 & 0.816 \end{bmatrix} * \begin{bmatrix} Y \\ x_v \\ y_v \end{bmatrix}$$

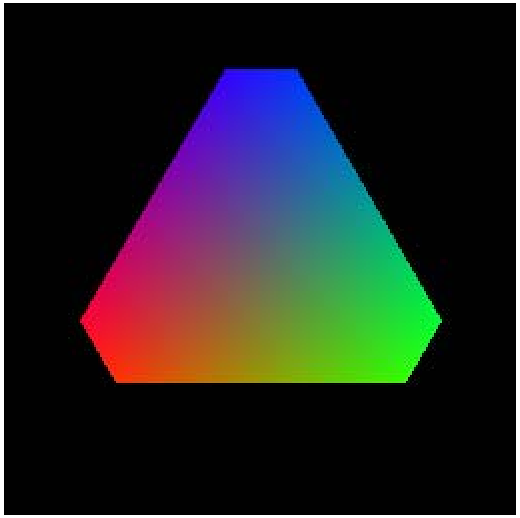
The Tour: Orthogonal Color Planes 1



Y=1/5

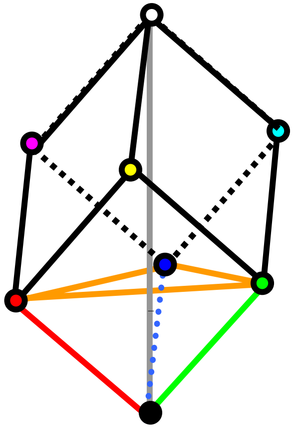


Y=1/3



Y=2/5

- The Y=1/3 plane has vertices defined by the pure red, green and blue points of the RGB cube.



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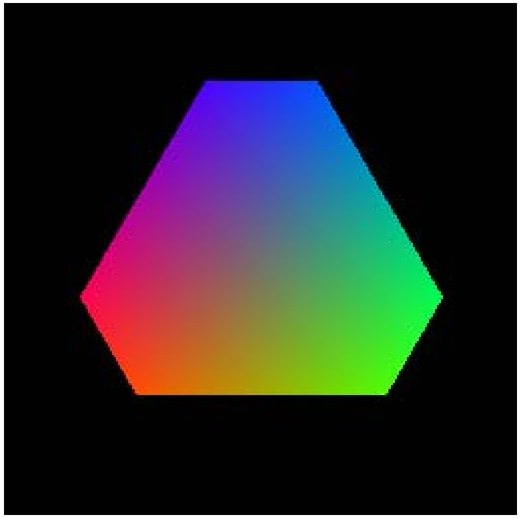
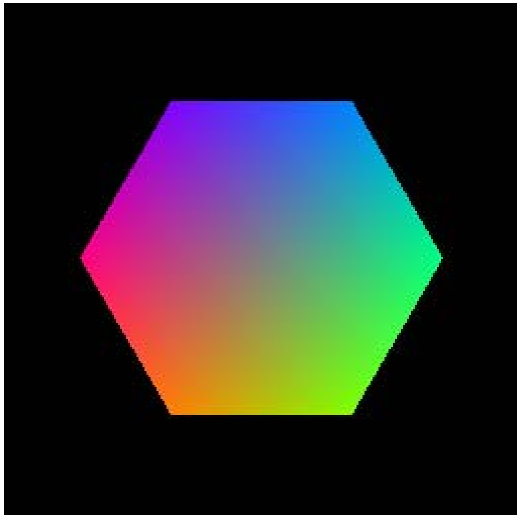
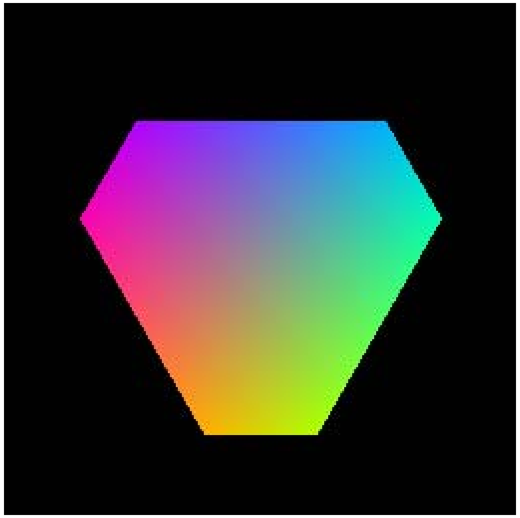
The Tour: Orthogonal Color Planes 2



Y=7/16

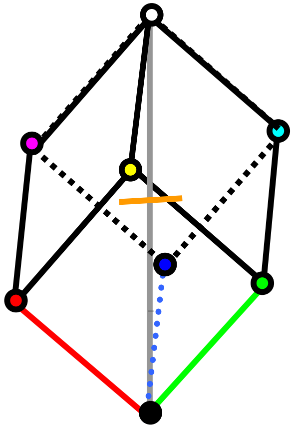


Y=1/2

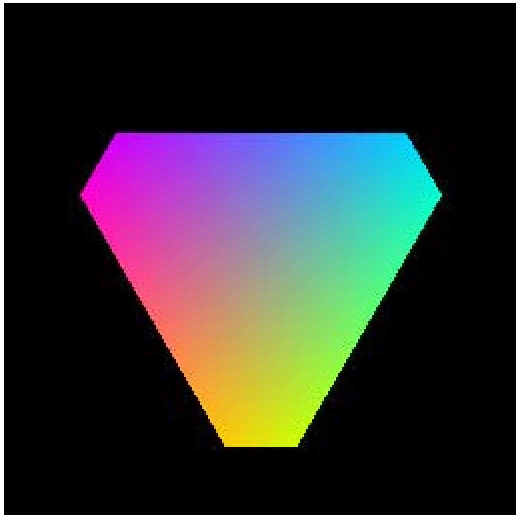


Y=9/16

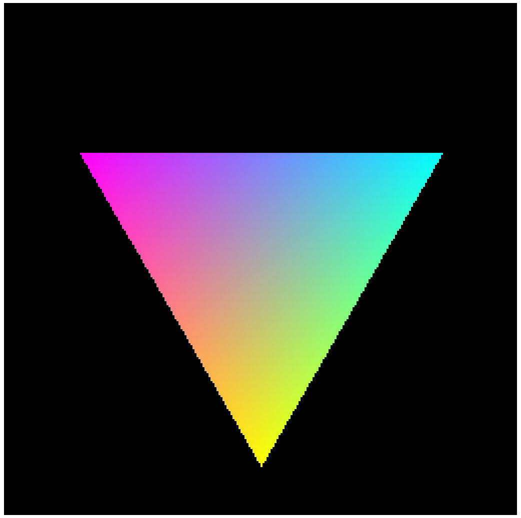
- The Y=1/2 plane is an equilateral hexagon.
- The edges intersect all six sides of the RGB cube.



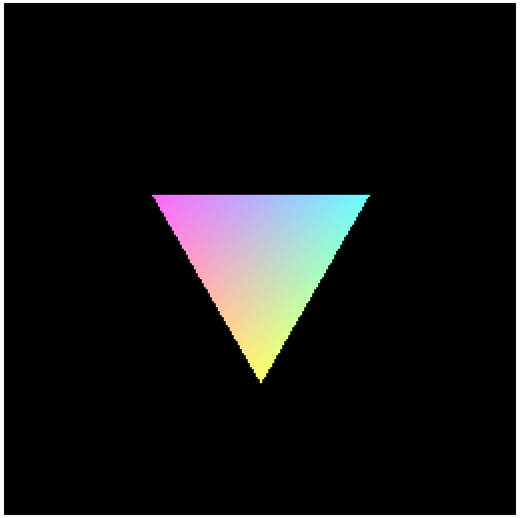
The Tour: Orthogonal Color Planes 3



Y=3/5

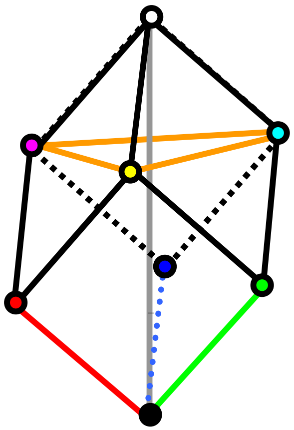


Y=2/3



Y=4/5

• The Y=2/3 plan vertices are defined by the pure cyan, yellow and magenta points of RGB cube.



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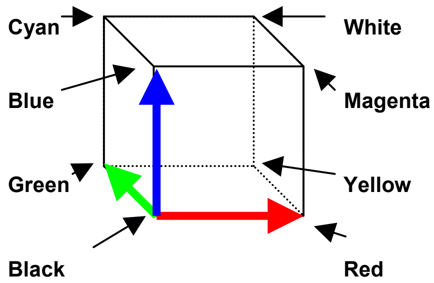
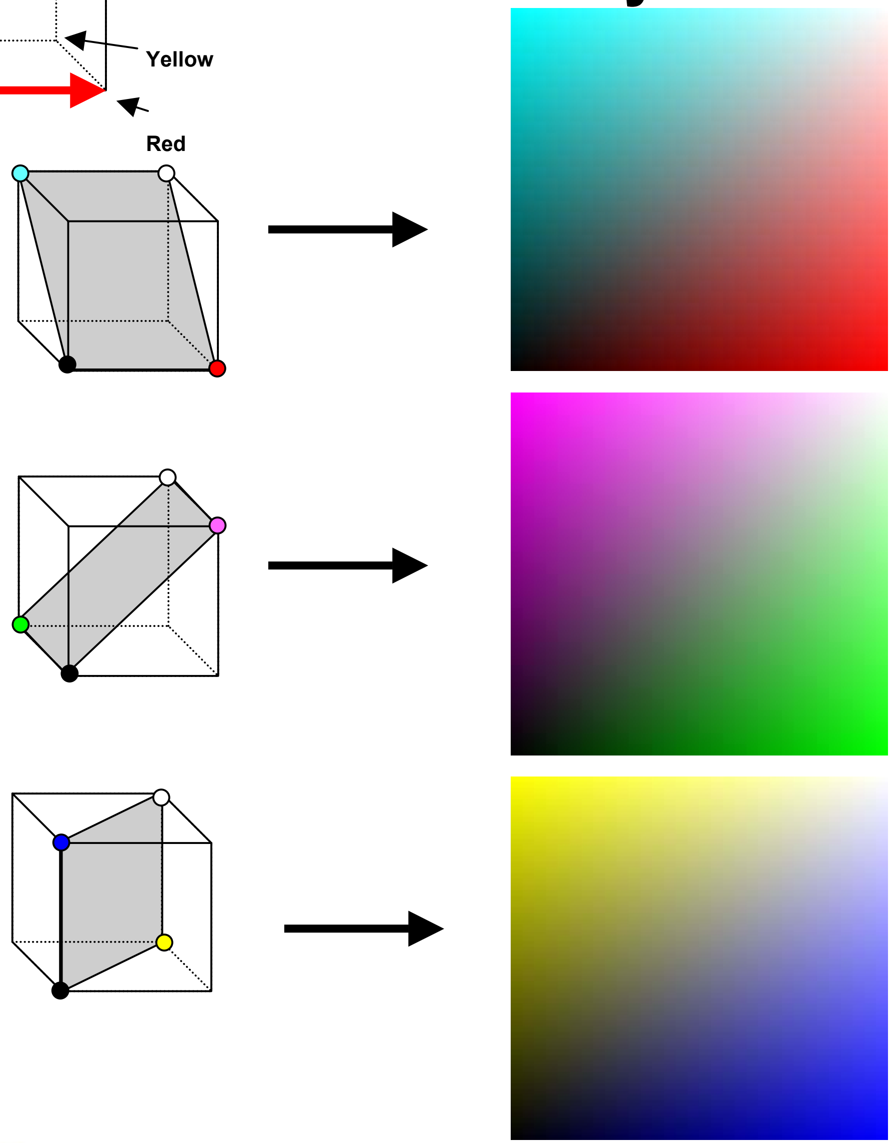
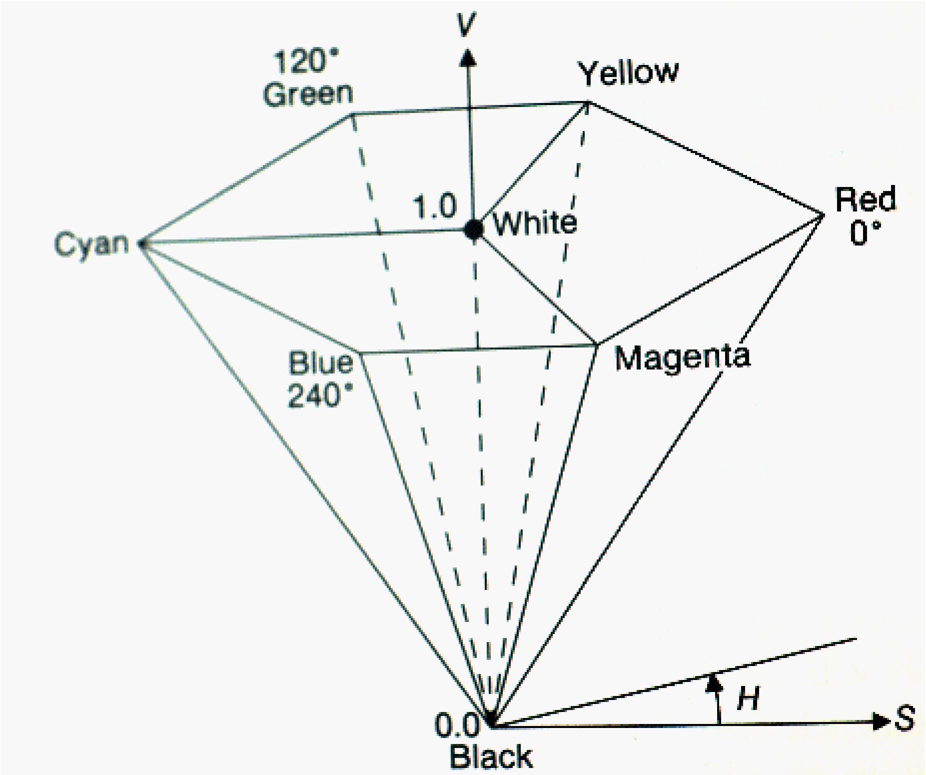
Three Planes That Bisect The RGB Cube That Fully Contain The Gray Vector



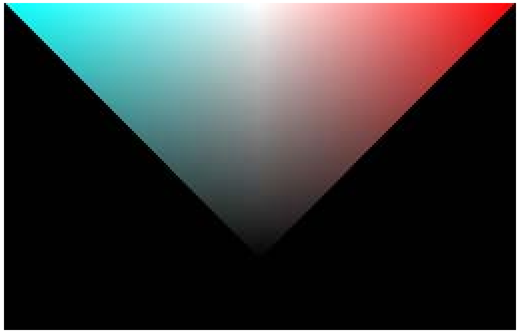


- Side views through gray vector.
- Indicates color symmetry about gray vector.
- Planes are flipped and rotated for presentation.
- Diagonal edge of plane is scaled to one from sqrt(2) for presentation.

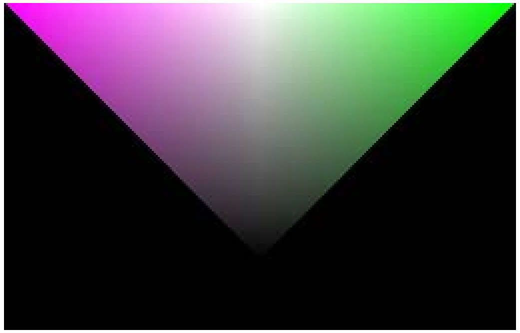
Traditional HSV Hexcone and Color Cross Sections



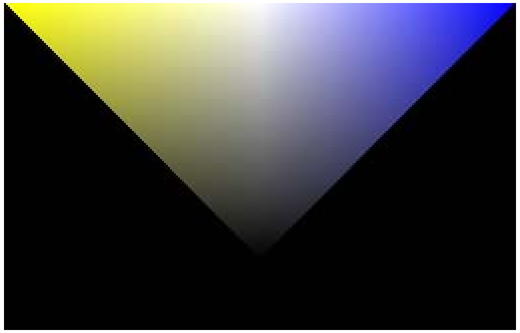
- Primary and complementary colors shown at 60 degree intervals of hue (H).
- Observe the concentration of white at top of slices.
- Complementary colors about V axis.
- V axis is conceptually along the gray vector.



Slice of HSV at H = 0° and 180°



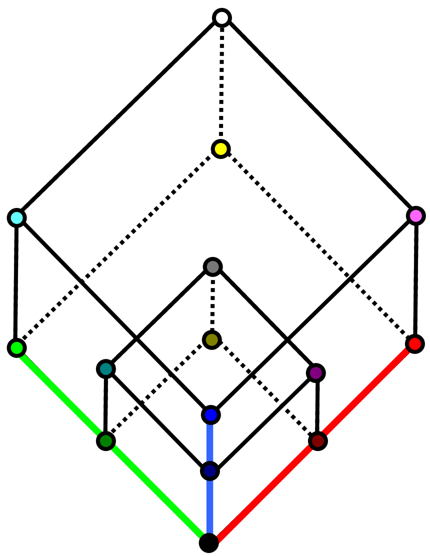
Slice of HSV at H = 120° and 300°



Slice of HSV at H = 240° and 60°

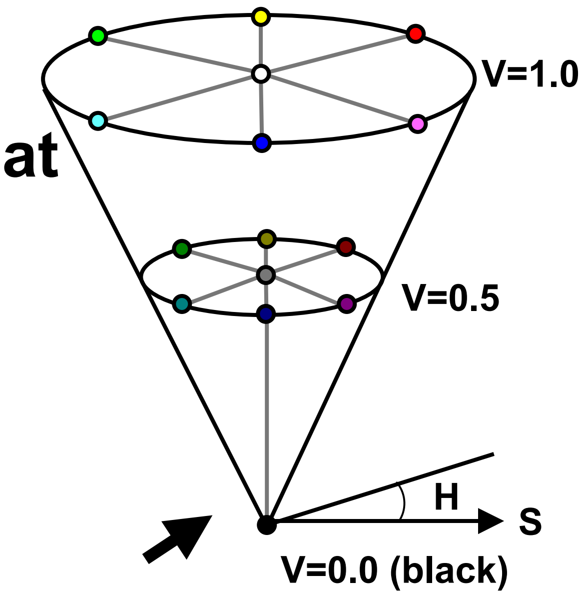
HSV Color Model

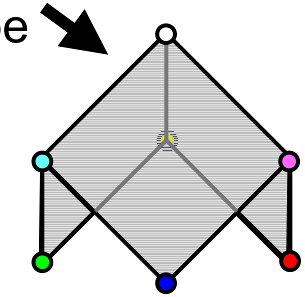
HSV is a color space that rotates and skews (in a non-linear fashion) the RGB color cube into an HSV cone.



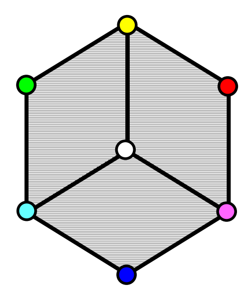
RGB color cube

HSV color space

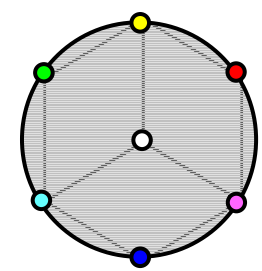




V=1.0 surface of RGB cube

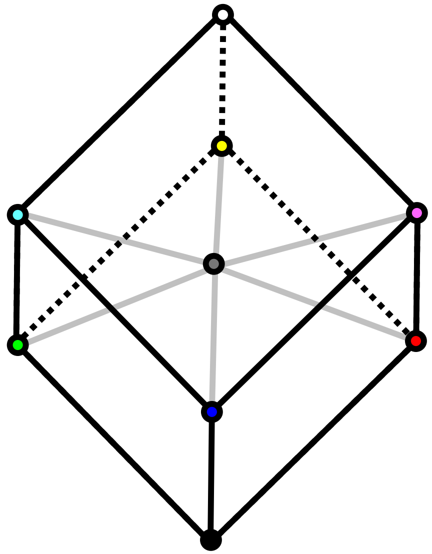


V=1.0 surface (viewed from above)



V=1.0 surface (flattened into 2-D and stretched into circular plane)

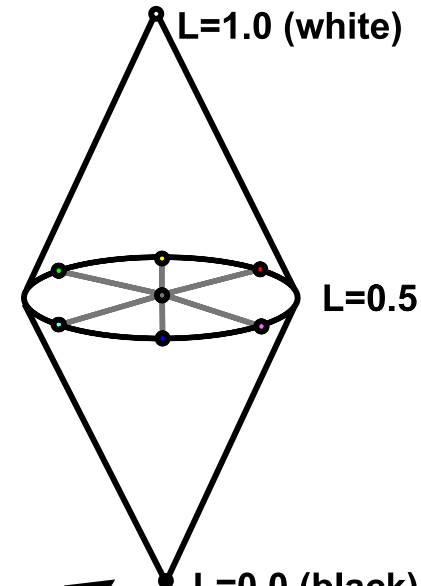
HLS Color Model

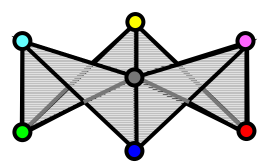


RGB color cube
(including "half-gray" point
R=G=B=0.5)

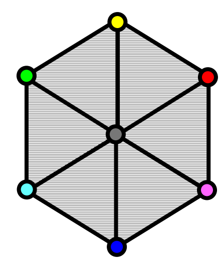
HLS is a color space that rotates and stretches (in a non-linear fashion) the RGB color cube into an HLS double cone.

HLS Color Space

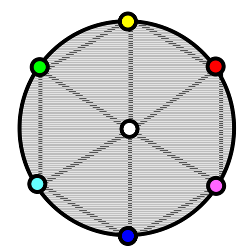




L=0.5 surface
of RGB cube



L=0.5 surface of
RGB cube (viewed
from above)



L=0.5 surface of RGB cube
(flattened into 2-D and
stretched into circular plane)

Recommendation 601 Component Video Standard

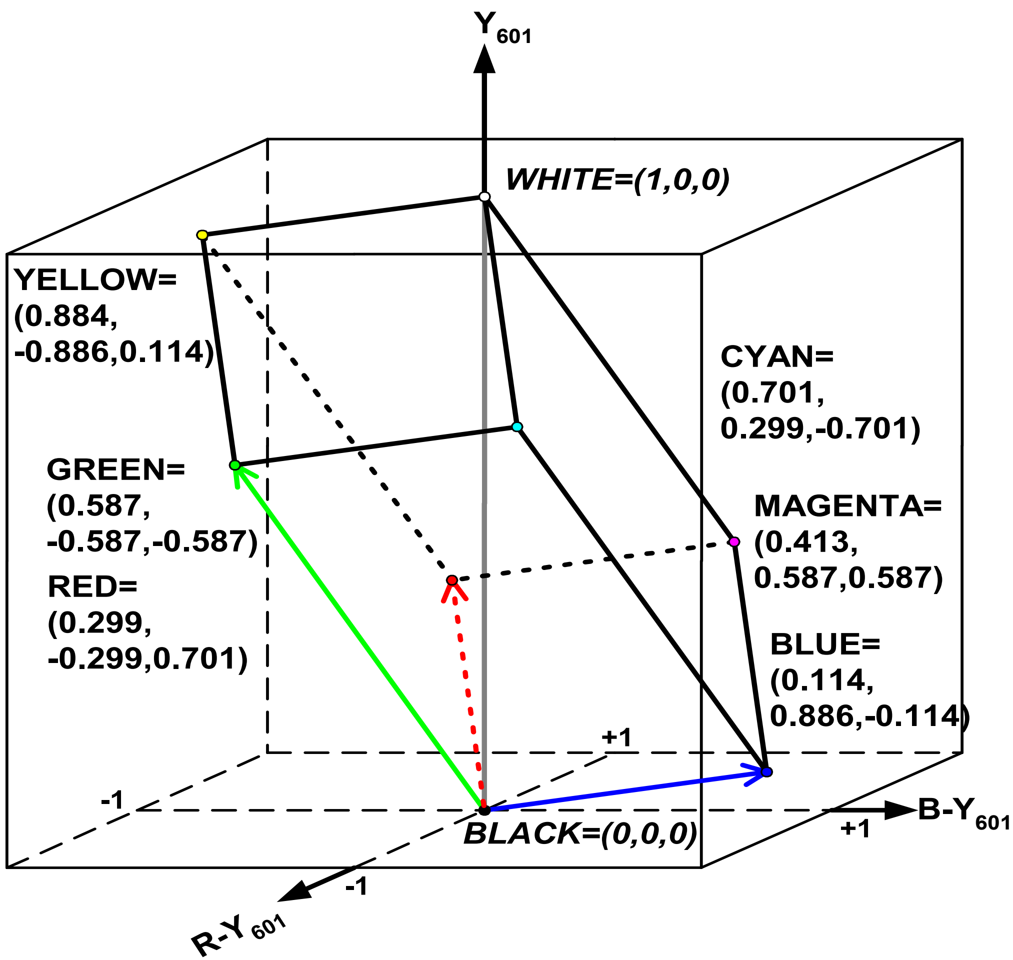
- **Reason for component video coding is to reduce bandwidth.**
- Encode RGB color cube representation into luminance (Y_{601}) and two color difference components ($B-Y_{601}$),($R-Y_{601}$) using

$$\begin{bmatrix} Y_{601} \\ B-Y_{601} \\ R-Y_{601} \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ -0.299 & -0.587 & 0.886 \\ 0.701 & -0.587 & -0.114 \end{bmatrix} * \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

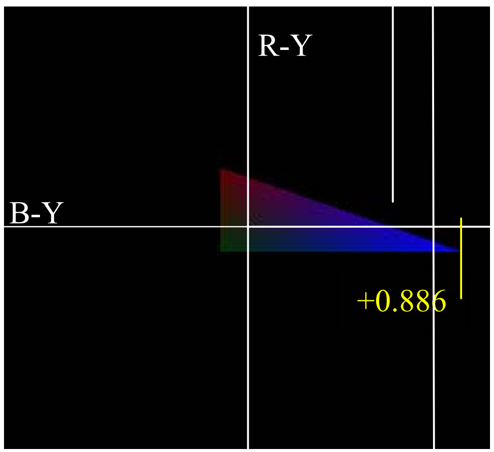
- **Color difference components are down-sampled to reduce bandwidth (or data capacity) of color video signal with minimal degradation noticed by viewer.**
- Green dominates human visual sensitivity so it is weighted greatest in luminance computation. So red/blue difference components are sub-sampled.
- Note RGB is assumed to be the R´G´B´ gamma corrected values.
- Y_{601},($B-Y_{601}$),($R-Y_{601}$) is scaled to 8 bits codes: Y_{601},C_B,C_R.
- Y_{601},C_B,C_R is 4:2:2 sampled; C_B,C_R sampled at half the rate of Y_{601}.

The Tour: Y_{601}, $(B-Y_{601})$,$(R-Y_{601})$ Color Space

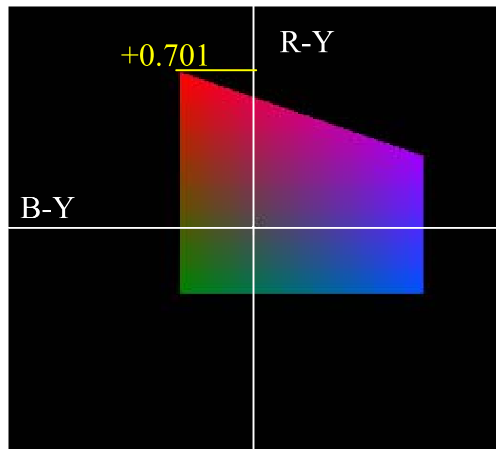
(RGB Color Cube Mapped Inside Y_{601}, $B-Y_{601}$, $R-Y_{601}$)



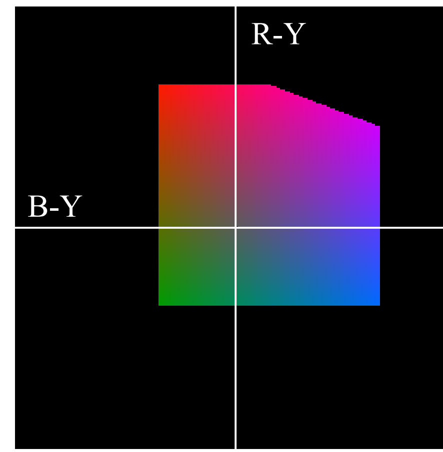
The Tour Up The Luminance Vector: Orthogonal Color Planes 1



Y_{601}=0.114 (Blue)

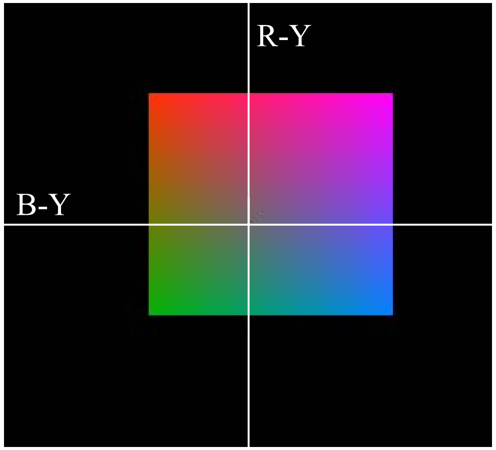


Y_{601}=0.299 (Red)

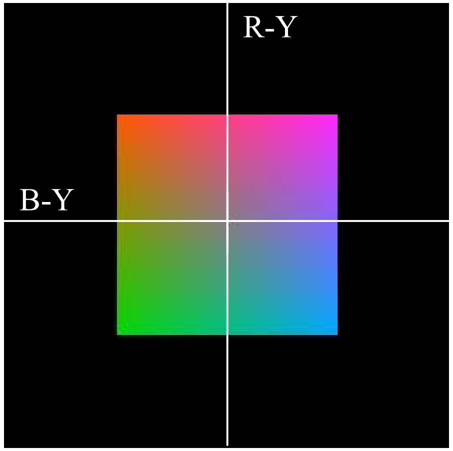


Y_{601}=0.356 (5 Sided)

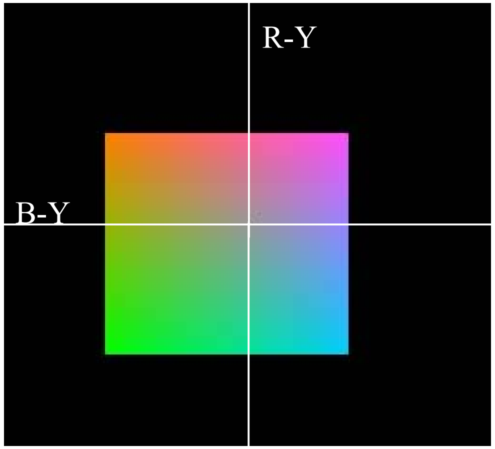
The Tour Up The Luminance Vector: Orthogonal Color Planes 2



Y_{601} =0.413 (Magenta)

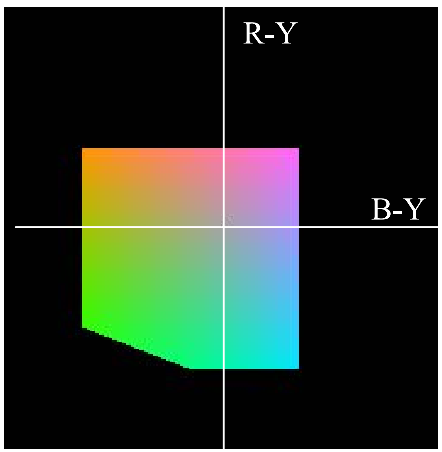


Y_{601} =0.5 (Half Way)

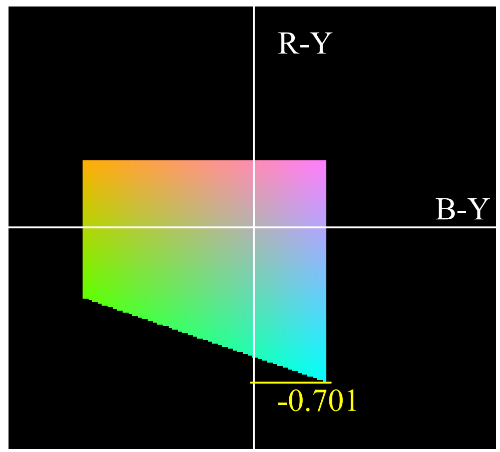


Y_{601} =0.587 (Green)

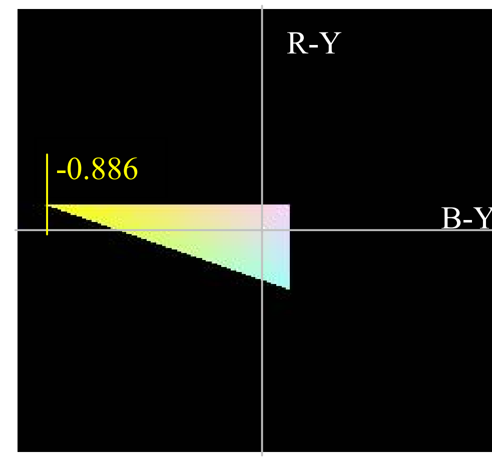
The Tour Up The Luminance Vector: Orthogonal Color Planes 3



Y_{601} =0.644 (5 Sided)

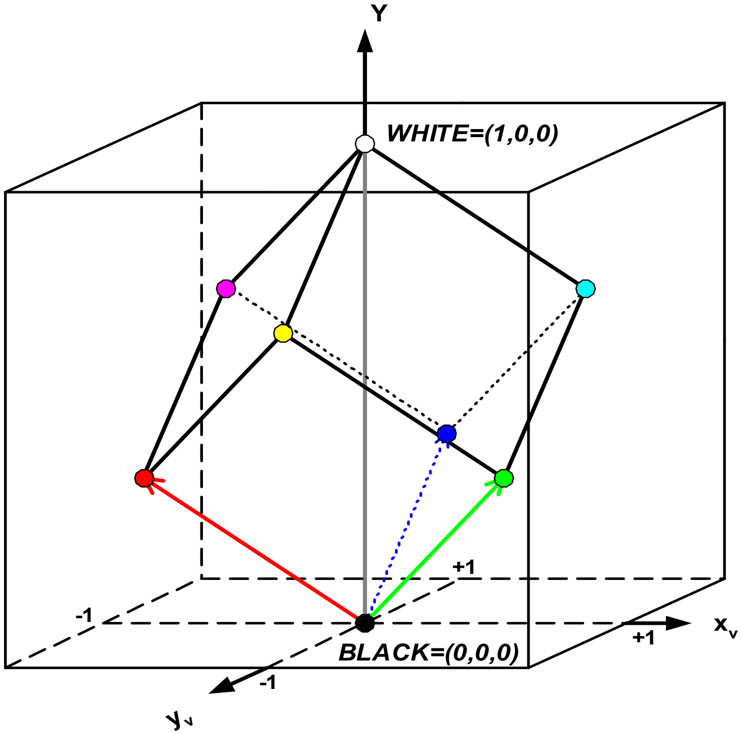


Y_{601} =0.701 (Cyan)

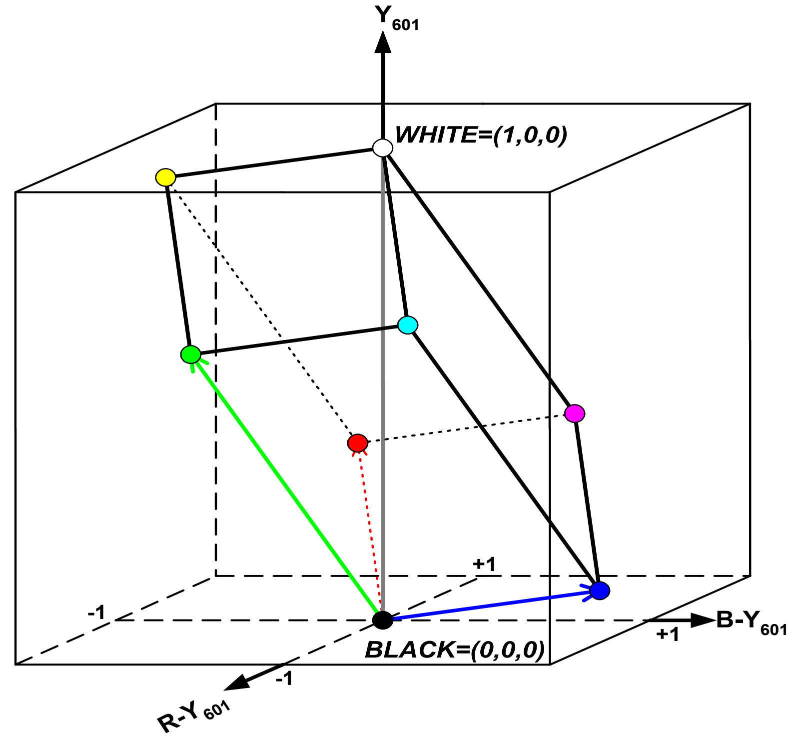


Y_{601} =0.884 (Yellow)

Comparison of Y, x_v, y_v Space to Y_{601}, B-Y_{601}, R-Y_{601} Space.



RGB Color Cube Mapped Inside Y, x_v, y_v Color
(RGB Cube Volume = 0.577)



RGB Color Cube Mapped Inside Y_{601}, B-Y_{601}, R-Y_{601} Color
(RGB Cube Volume = 0.587)

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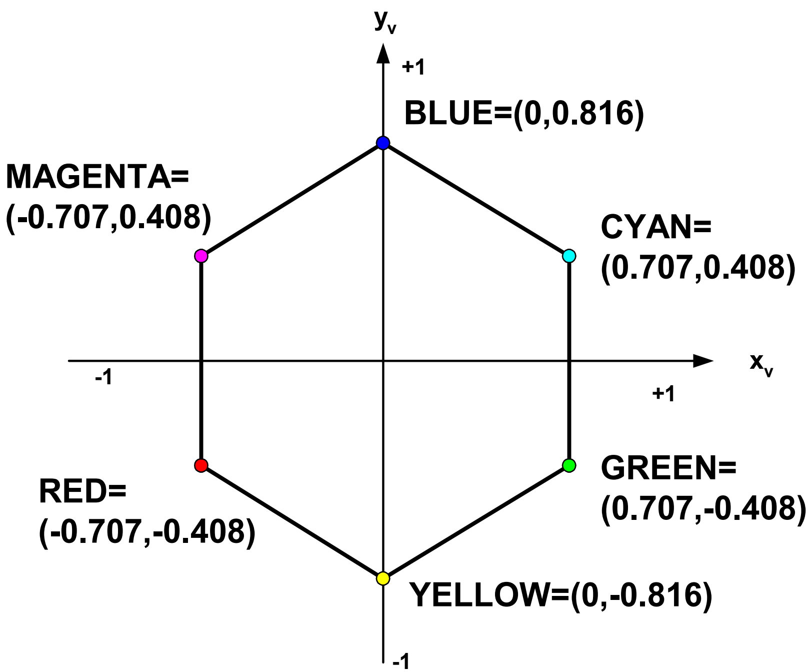
Relationships

- Relationship between the Y, x_v, y_v space and Y_{601},(B-Y_{601}),(R-Y_{601}) is

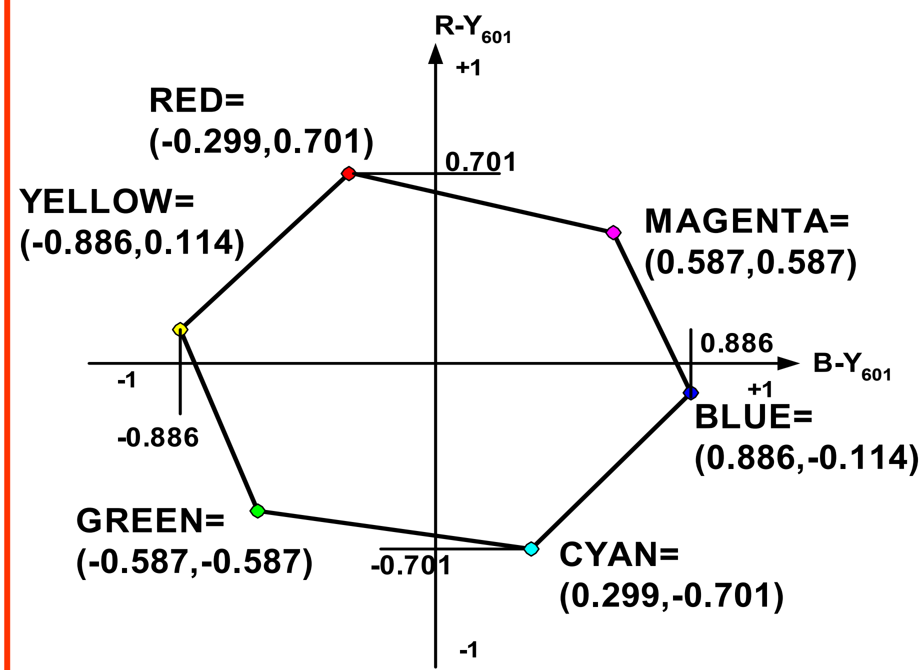
$$\begin{bmatrix} Y_{601} \\ B - Y_{601} \\ R - Y_{601} \end{bmatrix} = \begin{bmatrix} 1 & 0.204 & -0.269 \\ 0 & -0.204 & 1.084 \\ 0 & -0.911 & -0.140 \end{bmatrix} * \begin{bmatrix} Y \\ x_v \\ y_v \end{bmatrix}$$

- **Y_{601} is equivalent to scaled and gamma corrected gray vector (Y).**
- Volume of the valid RGB color region in each color space is approximately a quarter of region defined by cube of minimum to maximum color extent *[0.25 for Y, x_v, y_v , 0.237 for Y_{601},(B-Y_{601}),(R-Y_{601})].*
- Perceptually the color dynamic range is preserved when down-sampled (e. g. artifacts from sub-sampling chroma components are not obviously visible to viewer).

Primary and Complimentary Color Projections



x_v, y_v Components



$(B\text{-}Y_{601}),(R\text{-}Y_{601})$ Components

- Rotation of x_v, y_v is 97.35 degrees clockwise w.r.t. $(B\text{-}Y_{601}),(R\text{-}Y_{601})$.
- Original RGB space rotated a view coordinate system about the blue axis 45 degrees and then about the red axis 45 degrees up.

Summary

- Insightful tours up the gray scale vector (RGB space) and the luminance vector [Y_{601},(B-Y_{601}),(R-Y_{601}) space] were presented.
- These tours yielded interesting color images of planes orthogonal to the principle axis (either Y or Y_{601}).
- The symmetrical nature of these color coordinate systems is realized.
  - half way up the gray vector the color plane is an equilateral hexagon.
  - half way up the luminance vector the color plane is a perfect square.
- HSV (or HLS) are non-linear mappings from the RGB cube.
- The Y, x_v, y_v space is compared to Y_{601},(B-Y_{601}),(R-Y_{601}) space.
  - Y_{601} =Y
  - Colors rotated > 90 degrees.

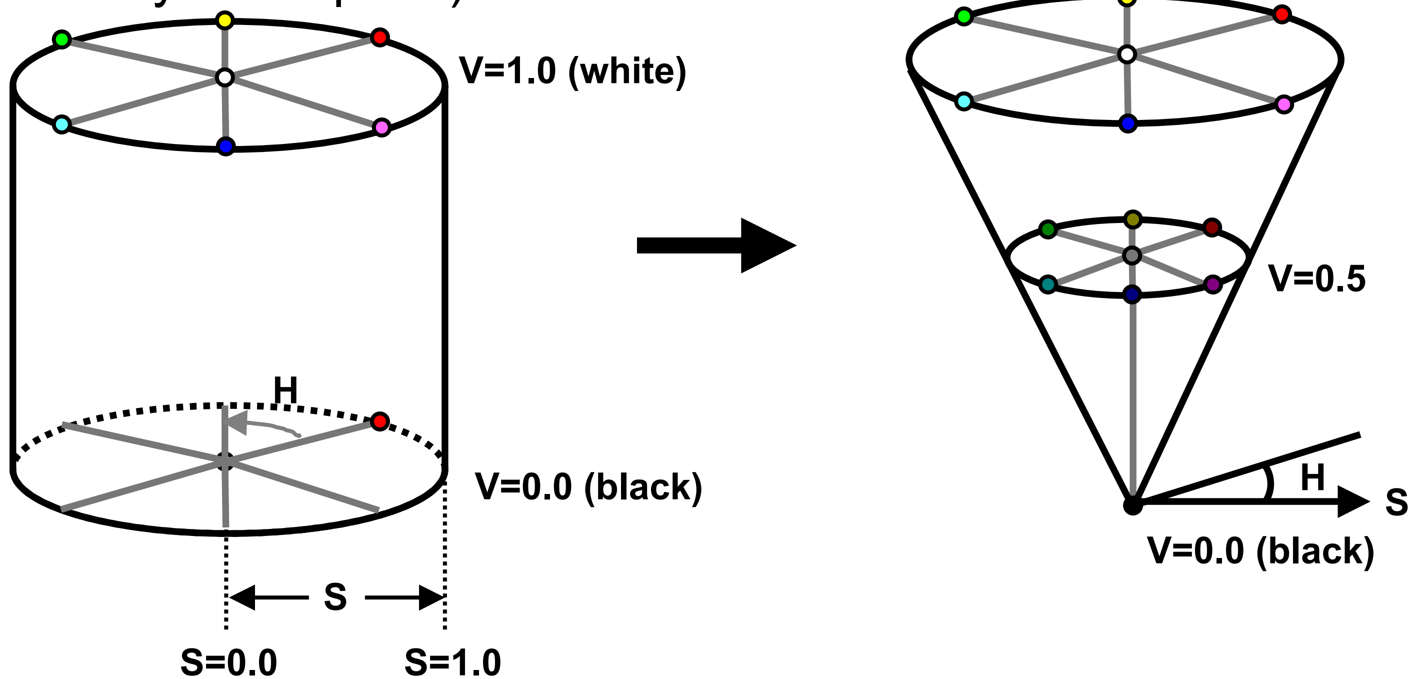
THANK YOU!

HOPE YOU ENJOYED THE TOUR.

Why the Hexacone Representation is Misleading!!!

True HSV Color Space
(Actually HSI Space)

Actual HSV Color Space



$$S'' = S' * (\sqrt{3} / (2 * \cos(abs((H \bmod 60) - 30.0))))$$

$$S' = S * V$$

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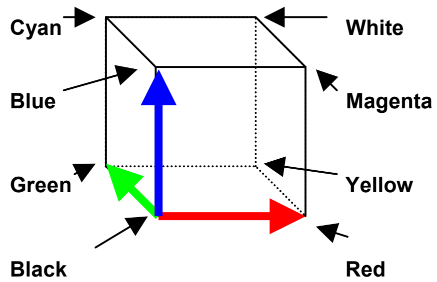
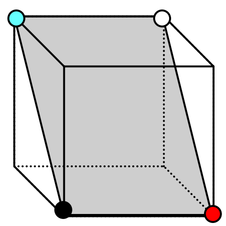
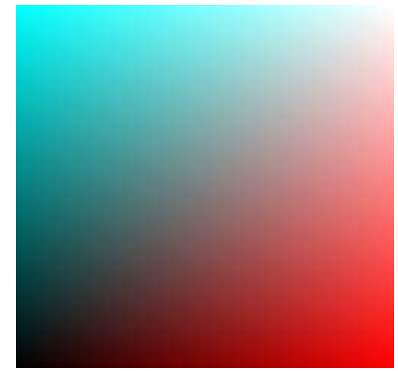
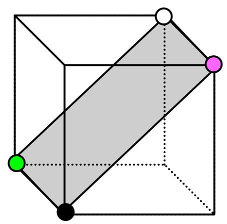


Three Planes That Bisect The RGB Cube That Fully Contain The Gray Vector

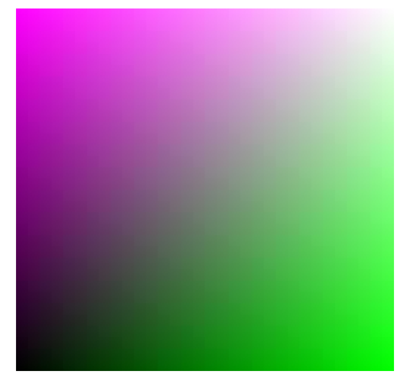


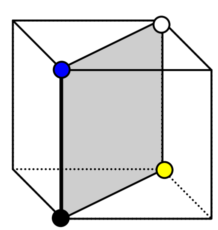
$$R(x_v,x_v) = x_v$$
$$G(x_v,x_v) = y_v$$
$$B(x_v,x_v) = y_v$$



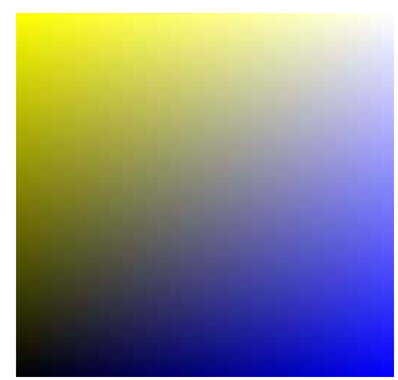


$$R(x_v,x_v) = y_v$$
$$G(x_v,x_v) = x_v$$
$$B(x_v,x_v) = y_v$$

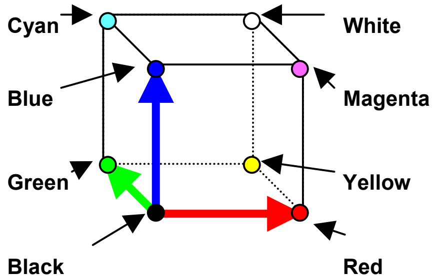




$$R(x_v,x_v) = y_v$$
$$G(x_v,x_v) = y_v$$
$$B(x_v,x_v) = x_v$$



RGB Color Space



RGB to HSV Conversion Equations

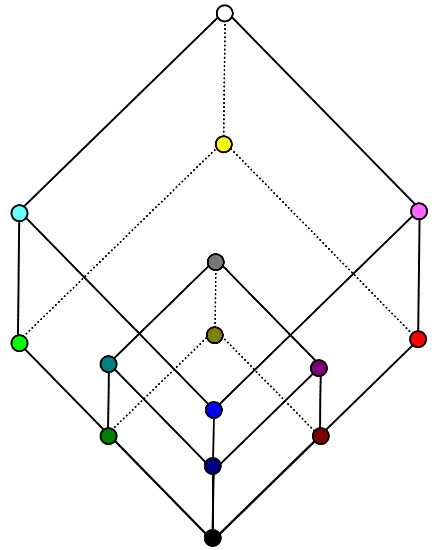
Min = min(R,G,B)
Max = max(R,G,B)

H' = (G-B)/(Max-Min) (for Max = R)
= 2 + (B-R)/(Max-Min) (for Max = G)
= 4 + (R-G)/(Max-Min) (for Max = B)

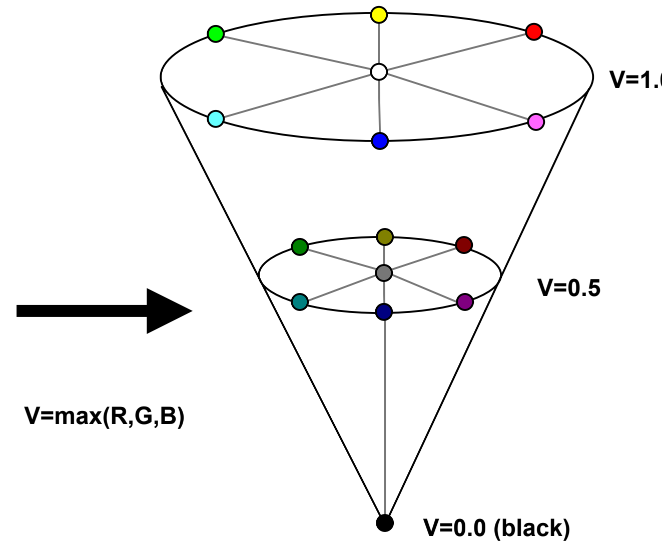
H = (60*H') mod 360
S = (Max-Min)/Max (for Max <> 0)
V = Max

If you are along the gray vector (Max=Min), S=0 and H is undefined.

RGB Color Space (rotated) with colors that have V=0.0 (black), V=0.5, and V=1.0

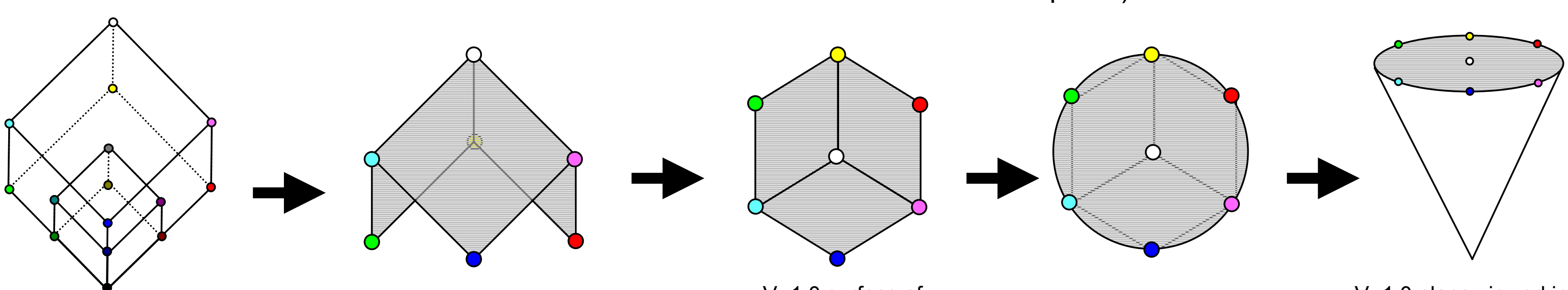


HSV Color Space



HSV is a color space that rotates and skews (in a non-linear fashion) the RGB color cube into an HSV cone.

Effectively, each circular plane that represents a value of V is a mapping of three sides of an RGB cube (turning a three-dimensional "half-cube" surface into a 2-D circular plane)



RGB color cube

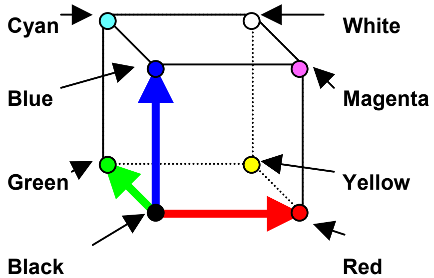
V=1.0 surface of RGB cube

V=1.0 surface of RGB cube (viewed from above)

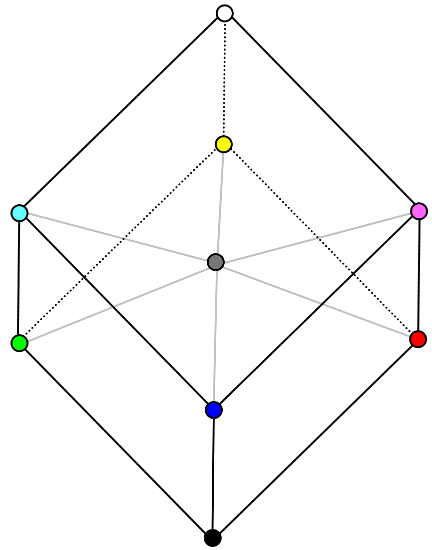
V=1.0 surface of RGB cube (flattened into 2-D and stretched into circular plane)

V=1.0 plane viewed in HSV cone

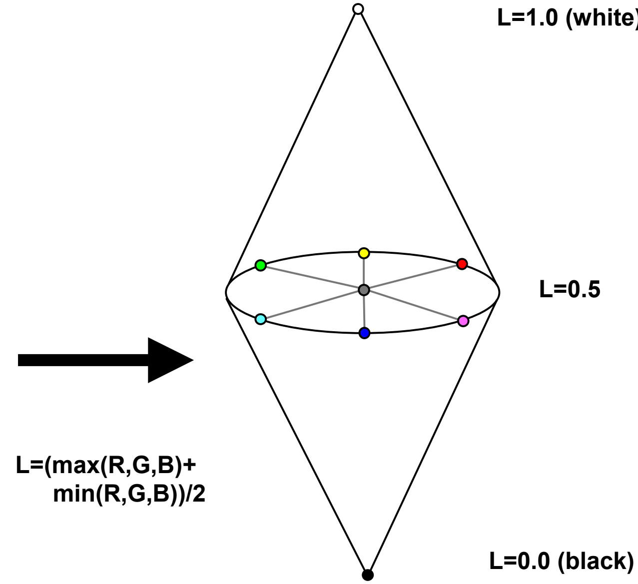
RGB Color Space



RGB Color Space (rotated) with colors that have L=0.0 (black), L=0.5, and L=1.0 (white)



HLS Color Space



RGB to HLS Conversion Equations

Min = min(R,G,B)
Max = max(R,G,B)

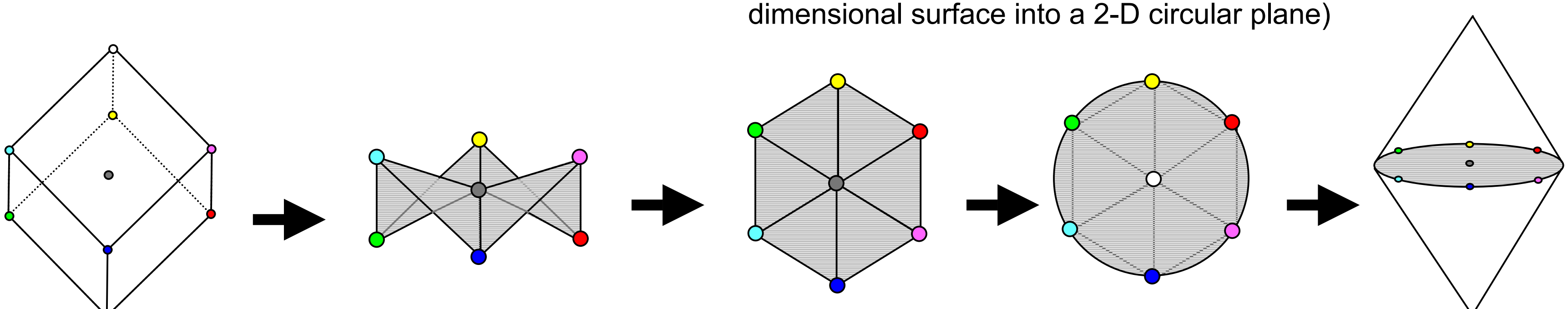
H' = (G-B)/(Max-Min) (for Max = R)
= 2 + (B-R)/(Max-Min) (for Max = G)
= 4 + (R-G)/(Max-Min) (for Max = B)

H = (60*H') mod 360
L = (Max+Min)/2
S = (Max-Min)/(Max+Min) (for L >= 0.5)
= (Max-Min)/(2-(Max+Min)) (for L < 0.5)

If you are along the gray vector (Max=Min), S=0 and H is undefined.

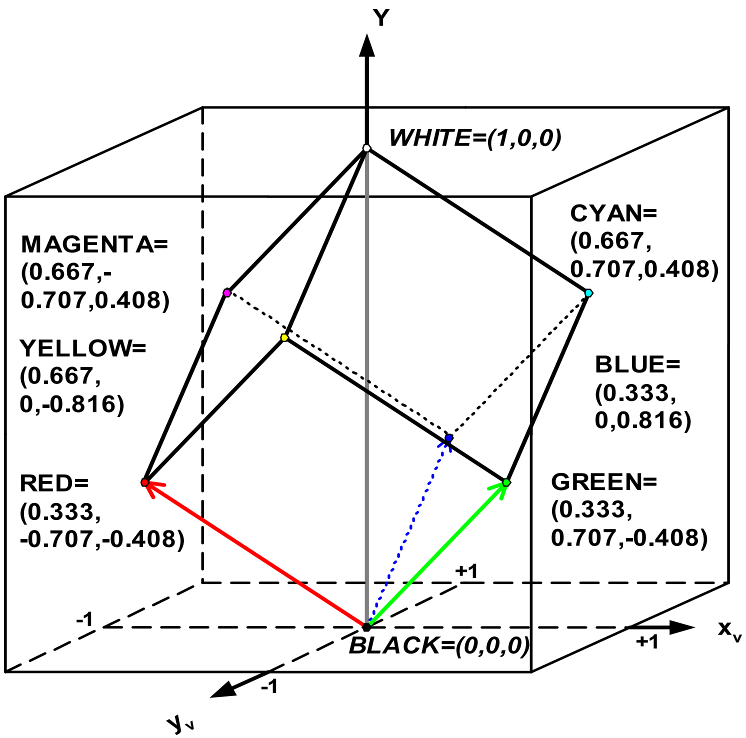
HLS is a color space that rotates and stretches (in a non-linear fashion) the RGB color cube into an dual-cone HLS space.

Effectively, each circular planar cut through the HLS "cone" that represents a value of L is a mapping of a irregular planar surface within the RGB cube (turning a three-dimensional surface into a 2-D circular plane)

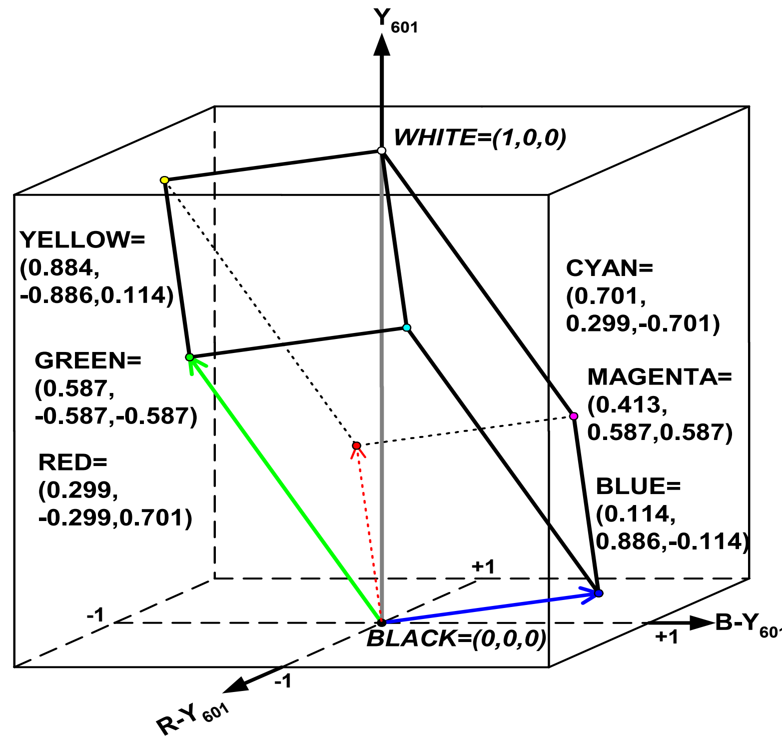


RGB color cube (including "half-grey" point R=G=B=0.5) | L=0.5 surface of RGB cube | L=0.5 surface of RGB cube (viewed from above) | L=0.5 surface of RGB cube (flattened into 2-D and stretched into circular plane) | L=0.5 plane viewed in HLS cone

Comparison of Y, x_v, y_v Space to Y_{601}, B-Y_{601}, R-Y_{601} Space.



RGB Color Cube Mapped Inside Y, x_v, y_v Color



RGB Color Cube Mapped Inside Y_{601}, B-Y_{601}, R-Y_{601} Color

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